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Cosmology from quantum gravity condensates

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Abstract

We analyze relationally evolving and effectively interacting Group Field Theory (GFT) models in the context of the GFT quantum gravity condensate analogue of the Gross-Pitaevskii equation for real Bose-Einstein condensates (BEC). More precisely, we firstly study the expectation value of the volume operator imported from Loop Quantum Gravity (LQG) in an isotropic restriction in the static case of a free and then interacting condensate system. In both cases one finds a nonvanishing condensate population for which the expectation value is dominated by the lowest nontrivial configurations of the quantum geometry. This indicates that the condensate consists of many smallest building blocks giving rise to an effectively continuous geometry, which suggests the interpretation of the condensate phase to correspond to a geometric phase. In a second step, we study the relational evolution of the such condensate systems with respect to a relational clock and demonstrate that from their effective dynamics the classical Friedmann equation can be recovered, thus reproducing and generalizing earlier obtained results by other authors.

Introduction

The most difficult problem for all quantum gravity approaches using discrete and quantum pregeometric structures is the recovery of continuum spacetime, its geometry, diffeomorphism invariance and General Relativity as an effective description for the dynamics of the geometry in an appropriate limit. It has been suggested, that a possible way of how continuum spacetime and geometry could emerge from a quantum gravity substratum in such theories is by means of at least one phase transition from a discrete pregeometric to a continuum geometric phase. One refers to such a process as "geometrogenesis" [1]. A particular representative in this class of approaches where such a scenario has been proposed is GFT [2] where one tries to identify the continuum geometric phase to a potential condensate phase of the underlying quantum gravity system [3] with a tentative cosmological interpretation [4, 5, 6, 7, 8, 9].

GFT condensates

GFTs are Quantum Field Theories defined over group manifolds and are characterized by their combinatorial nonlocal interaction terms. In models for $4d$ quantum gravity, the central object of the theory is most generally a complex-valued scalar field

$$\varphi : G^4 \to \mathbb{C},$$

where G is the local gauge group of gravity. Here we assume $G = \mathrm{SU}(2)$ which is the local gauge group of Ashtekar-Barbero gravity. Using the prescription of GFT as a 2nd quantization of LQG, the field is promoted to a field operator where $\hat{\varphi}^\dagger$ excites a quantum of space over the Fock vacuum $|\emptyset\rangle$, as illustrated in Fig. 1.

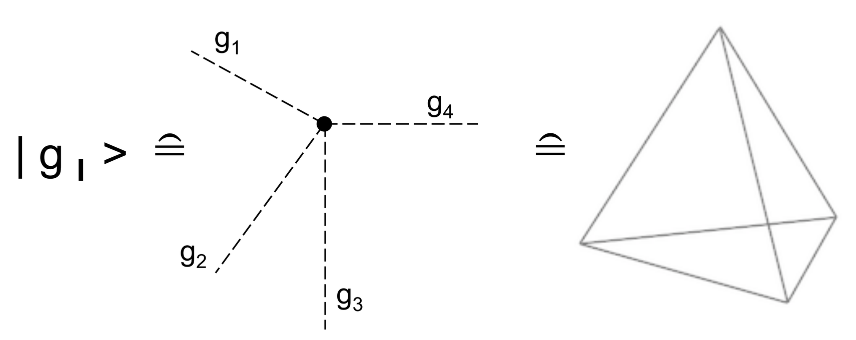

Figure 1: A GFT field quantum corresponds to a quantum tetrahedron or simply a chunk of space.

The quantum dynamics of the field is then encoded by the partition function

$$Z_{\mathrm{GFT}} = \int [D\varphi][D\bar{\varphi}] e^{-S[\varphi,\bar{\varphi}]}$$

with the action

$$S = \int\int (dg)^4 (dg')^4 \bar{\varphi}\ \mathcal{K}(g_I, g'_I)\ \varphi + \mathcal{V}[\varphi(g_I), \bar{\varphi}(g_I)],$$

where $\mathcal{K}$ is a local kinetic operator and $\mathcal{V}$ denotes a combinatorial nonlocal interaction.

Due to such interactions, in the perturbative expansion of Z_{GFT} the Feynman diagrams of the theory are dual to cellular complexes. Depending on $\mathcal{K}$ and $\mathcal{V}$ and thus on the details of the Feynman amplitudes, the sum over the latter can give rise to a discrete definition of the covariant path integral for $4d$ quantum gravity.

The possible occurrence of a phase transition in such systems [3] is highly interesting, since it has been suggested that phase transitions from a symmetric to a condensate phase for such GFT models could be a realization of the above-mentioned geometrogenesis scenario, as illustrated by Fig. 2.

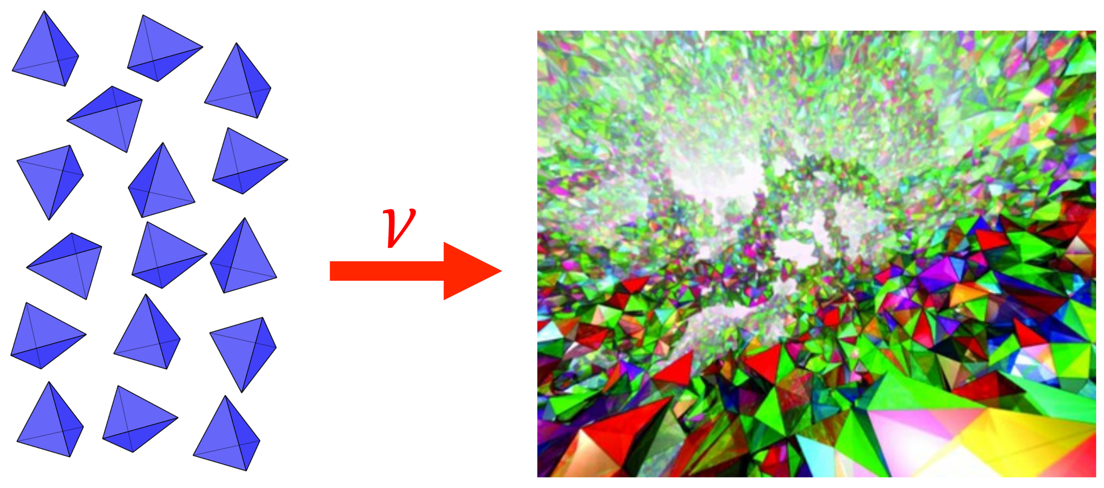

Figure 2: Illustration of the geometrogenesis scenario. (Right: © MPI for Gravitational Physics)

In this context, the central assumption of GFT Condensate Cosmology is that the suggested continuum geometric phase of a particular GFT model is ideally approximated by a condensate state given by

$$|\sigma\rangle = \mathcal{N}(\sigma) e^{\int (dg)^4 \sigma(g_I) \hat{\varphi}^\dagger(g_I)} |\emptyset\rangle, \quad \langle\sigma|\hat{\varphi}(g_I)|\sigma\rangle = \sigma(g_I),$$

where $\sigma(g_I)$ is called mean field. Its dynamics are obtained from

$$\left\langle \frac{\delta S[\varphi, \bar{\varphi}]}{\delta \bar{\varphi}} \right\rangle_{\varphi=\sigma} = 0,$$

which gives rise to the analogue of the Gross-Pitaevskii equation for real BECs. Such states are suitable to describe spatially homogeneous universes [4].

With an appropriately motivated kinetic term and when including a free massless scalar field ϕ serving as a relational clock, for a real-valued GFT field one is left with studying the solutions to the condensate equation of motion

$$(\tau \partial_\phi^2 - \sum_{I=1}^{4} \Delta_{g_I}) \sigma(g_I, \phi) + m^2 \sigma(g_I, \phi) + \frac{\delta \mathcal{V}}{\delta \sigma} = 0,$$

where $\tau > 0$ holds and $m^2 < 0$ indicates that we are in the condensate phase.

Static and isotropic case

In a first step, we consider condensates in an isotropic restriction which are also static with respect to the clock ϕ and are either free or only locally interacting [8]. With this the equation of motion is recast into

$$[-\Delta_{S^3} + 2\mu]\sigma(\psi) + 2\frac{\delta \mathcal{V}[\sigma]}{\delta \sigma} = 0, \quad \mu \equiv \frac{m^2}{12}, \quad \psi \in [0, \frac{\pi}{2}].$$

Using Fourier analysis on $\mathrm{SU}(2)$, its solutions can be decomposed via

$$\sigma(\psi) = \sum_{m \in \mathbb{N}_0/2} (2m+1) \chi_m(\psi) \sigma_m$$

with respect to the plane waves given in terms of the group characters $\chi_m(\psi)$. Using this, the expectation value of the LQG volume operator can be decomposed as

$$\langle \hat{V} \rangle \equiv V = \ell_p^3 \sum_{m \in \mathbb{N}_0/2} \sigma_m^2 V_m, \quad \text{with} \quad V_m \sim m^{3/2}.$$

In either case, the largest contributions to V stem from the lowest ranging modes m, as illustrated by Fig. 3, which indicates that the condensate indeed consists of many smallest building blocks constituting an effectively continuous homogeneous and isotropic quantum geometry if their number N is large [8].

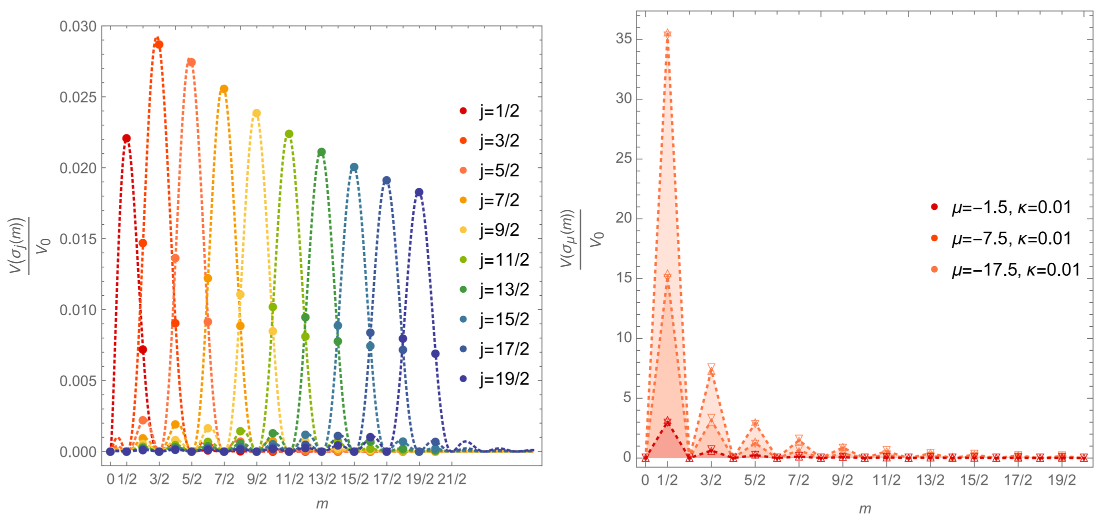

Figure 3: Discrete spectrum of the volume operator (in arbitrary units) with respect to the solutions $\sigma(\psi)$ of the free (left, wherein j denotes eigenmodes of $-\Delta_{S^3}$) and an interacting system with effective potential $V[\sigma] = \frac{\mu}{2}\sigma^2 + \frac{\kappa}{4}\sigma^4$ for which $\kappa > 0$ holds.

Evolving and isotropic case

When including the evolution with respect to the clock ϕ, as in [8], in the free case it is possible to show that the condensate quickly settles dynamically into the lowest nontrivial configuration of the quantum geometry, as

$$\lim_{\phi \to \pm\infty} V(\phi) \sim \ell_p^3\ V_{1/2}\ e^{\pm 2\sqrt{\frac{\mu + \frac{3}{2}}{-\tau}}\phi}.$$

Exploiting this fact, we can recover the classical Friedmann equation

$$H^2 = \left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}$$

written in terms of the relational clock ϕ, thus generalizing earlier obtained results by Oriti, Sindoni and Wilson-Ewing [5, 6]. Interactions in general lead to a recollapse as long as they are bounded from below [8, 9].

Conclusions and outlook

Using the GFT approach to quantum gravity, it is possible to construct effectively continuous as well as homogeneous and isotropic quantum geometries whose effective dynamics gives rise to the classical Friedmann equation in the semiclassical limit.

This work could be extended in many ways. For instance, the effect of nonlocal interactions and other gauge groups such as the Lorentz group has to be studied to clarify the geometric interpretation of the condensate and the nature of the suggested phase transition.

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